





3.3

Polynomial and Synthetic Division

$$f(x) = (6x^3 - 19x^2) + 16x - 4$$

$$x^2(\underline{6x-19}) + 4(\underline{4x-1})$$

Divide 12 by 3

$$= 4$$

$$12 = 3 \cdot 4$$

$$12 = 3 \cdot 2 \cdot 2$$

Use long division to divide:

$$\underline{34867} \div \underline{23}$$

$$\begin{array}{r} 1515 \text{ R } 22 \\ 23 \overline{) 34867} \\ \underline{-23} \\ 118 \\ \underline{-115} \\ 36 \\ \underline{-23} \\ 137 \\ \underline{115} \\ 22 \end{array}$$

$$(23) \left(1515 \frac{22}{23} \right)$$

If $(x-2)$ is a factor of
 $f(x) = 6x^3 - 19x^2 + 16x - 4$,
then $f(x) = (x - 2) \cdot \underbrace{q(x)}$

$12 = 3 \cdot ?$

Division Algorithm:

$$f(x) = (d(x))(q(x)) + r(x)$$

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

$$\frac{34867}{23} = 1515 + \frac{22}{23}$$

If $(x-2)$ is a factor of

$$f(x) = 6x^3 - 19x^2 + 16x - 4,$$

$$\text{then } f(x) = (x - 2) \cdot q(x)$$

$$\begin{array}{r} \overline{6x^2 - 7x + 2} \\ x-2 \overline{) 6x^3 - 19x^2 + 16x - 4} \\ \underline{-6x^3 - 12x^2} \\ -7x^2 + 16x \\ \underline{-7x^2 + 14x} \\ 2x - 4 \\ \underline{-2x - 4} \\ 0 \end{array}$$

$$(x-2)(6x^2 - 7x + 2)$$

$$(x-2)(3x-2)(2x-1)$$

Ex 1 Use Long Division

$$\begin{array}{r} x^2 - 3x + 1 \\ 4x + 5 \overline{) 4x^3 - 7x^2 - 11x + 5} \\ \underline{-4x^3 + 5x^2} \downarrow \\ \phantom{4x + 5 \overline{) }} -12x^2 - 11x \\ \underline{-12x^2 - 15x} \downarrow \\ \phantom{4x + 5 \overline{) }} 4x + 5 \\ \underline{4x + 5} \\ \phantom{4x + 5 \overline{) }} 0 \end{array}$$

$(4x + 5)(x^2 - 3x + 1)$

$$(5x^3 - 6x^2 + 8) \div (x - 4)$$

$$\begin{array}{r}
 \overline{5x^2 + 14x + 56} \\
 x-4 \overline{) 5x^3 - 6x^2 + 0x + 8} \\
 \underline{- 5x^3 - 20x^2} \quad \downarrow \\
 14x^2 + 0x \\
 \underline{- 14x^2 - 56x} \quad \downarrow \\
 56x + 8 \\
 \underline{- 56x - 224} \\
 232
 \end{array}$$

$$\boxed{5x^2 + 14x + 56 + \frac{232}{x-4}}$$

Shortcut to Long Division-- Synthetic Division

- 1) only applies when the divisor is $x - c$
- 2) when every descending power of x has a place in the dividend

$x+4$ yes
 $x-2$ yes

$3x-2$ NO

Ex 3 Do same problem using Synthetic Division

$\frac{1}{2}x+3$ NO

$(5x^3 - 6x^2 + 8) \div (x - 4)$

4

5 x^3 -6 0 8

↓ +20 +56 +224

5x² + 14x + 56

232 ← remainder

zero →

Use synthetic division to divide
 $-x^3 + 75x - 250$ by $x + 10$

$$\begin{array}{r|rrrr} -10 & -1 & 0 & 75 & -250 \\ & \downarrow & & & \\ & -1 & 10 & -100 & 250 \\ \hline & -1 & 10 & -25 & \boxed{0} \end{array}$$

$$(x+10)(-x^2 + 10x - 25)$$

$$- (x+10)(x^2 - 10x + 25)$$

$$\boxed{-(x+10)(x-5)^2}$$

Remainder Theorem: If a polynomial $f(x)$ is divided by $x - k$, the remainder is $r = f(k)$.

$$x^5 - 3x^3 + 8x - 11$$

$$f(4) = (4)^5 - 3(4)^3 + 8(4) - 11 = \underline{\underline{853}}$$

$$\begin{array}{r|rrrrrr} 4 & 1 & 0 & -3 & 0 & 8 & -11 \\ & & 4 & 16 & 52 & 208 & 864 \\ \hline & 1 & 4 & 13 & 52 & 216 & \boxed{853} \end{array}$$

Use the Remainder Theorem to find $f(2)$.

$$f(x) = x^3 - 2x^2 - 4x + 1$$

$$f(2) = -7$$

$$f(-1) = 2$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -4 & 1 \\ & \downarrow & & & \\ & 2 & 0 & -8 & \\ \hline & 1 & 0 & -4 & -7 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -4 & 1 \\ & \downarrow & & & \\ & -1 & 3 & 1 & \\ \hline & 1 & -3 & -1 & 2 \end{array}$$



HW: Pg 295 #7, 10, 14, 19,
28, 37, 45, 48,
49, 58, 70, 73